Eurasian Journal of Civil Engineering and Architecture

Volume 2 Issue 2

Effects of Soil-structure Interaction on Free Vibrations of Industrial Chimneys via Dynamic Stiffness Formulations

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ABSTRACT

In this study, free vibration analysis of an industrial chimney is investigated considering soil-structure interaction (SSI) using dynamic stiffness method (DSM). The chimney is modeled as a Timoshenko beam with constant cross-section. One rotational and one horizontal linear spring are used to model foundation flexibility for different soil conditions. The dynamic stiffness formulations are derived by using end forces and end displacements of chimney. Different soil shear modulus values are used to reveal their effects on free vibrations of chimneys considering SSI. The DSM results are tabulated with exact analytical solution results where a very good agreement is observed. Several mode shapes are plotted.

Keywords: Chimney, dynamic stiffness method, free vibration, soil-structure interaction.

INTRODUCTION

The dynamic analysis of chimneys takes an important place in civil engineering as they are essantial for industrial and nuclear facilities. Thus, the accuracy of free vibration analysis results of chimneys is an attractive research area. In general, the structures are modeled with classical support conditions such as fixed supports for simplicity. However, the fixed support assumption may cause incorrect results especially for structures on weak soil. Considering SSI provides realistic results for chimneys to be constructed on weak soil. Limited number of studies are concerned about free vibrations of tall chimneys in open literatures. Guler (1998) obtained natural frequencies and mode shapes of two high-rise chimneys that have variable cross-sections using Galerkin method. Chmielewski *et al.* (2005) obtained first four natural frequencies and mode shapes of a 245 m high chimney in Poland using finite element method (FEM). The analyses of the study is based on Kausel approach using linear rotational and horizontal springs. Górski (2015) investigated natural frequencies and damping ratio of a 295 m high chimney using GPS technology. Bozyigit, Ozturkoglu and Catal (2015) calculated first three natural frequencies and mode shapes of a chimney considering SSI by using differential transform method (DTM).

The importance of SSI is not limited for tall chimneys. There are numerous studies about vibrations of beams, retaining walls and frames considering SSI. Demirdag and Yesilce (2011) investigated free vibrations of Timoshenko beams with a tip mass using DTM. This study considers linear rotational spring at the column-foundation joint to reflect the foundation flexibility. Cacciola et al. (2015) researched vibration control of piled structures considering SSI. Doménech et al. (2016) investigated SSI on free vibration response of beams of railway bridges. Ramezani et al. (2016) calculated first three natural frequencies of a retaining wall according to Euler-Bernoulli beam theory (EBT) considering SSI.

The DSM is an effective tool for investigating vibrations of beams, shells, plates and beam assembly structures. The DSM provides exact solutions as the method uses exact mode shapes (Banerjee, 1997). The DSM was used by researchers for vibrations of different types of structures successfully (Bao-Hui et al. 2011, Banerjee 2012, Su and Banerjee 2015, Bozyigit and Yesilce 2016, Trong and Khiem 2017).

In this study, free vibration analysis of a chimney is performed via DSM considering SSI. One rotational and one translational linear springs are used to reflect the foundation flexibility. The chimney model is considered as a Timoshenko beam. Different soil shear modulus values are used to observe the effects on free vibration characteristics of chimney. The DSM results are tabulated with results of analytical solution. The mode shapes are plotted using Matlab.

MODEL AND FORMULATION

The free vibration analysis of Timoshenko element chimney is performed according to assumptions below:

1. The material of chimney is homogenous and isotropic.
2. The chimney behaves linear and elastic.
3. Axial vibrations are neglected.
4. Damping is neglected.

The chimney model is presented in Figure 1 where H is height of the chimney, x,y and z are axes, R_0 is radius of foundation of chimney, k_x and k_θ are stiffness values of translational and rotational springs, respectively.

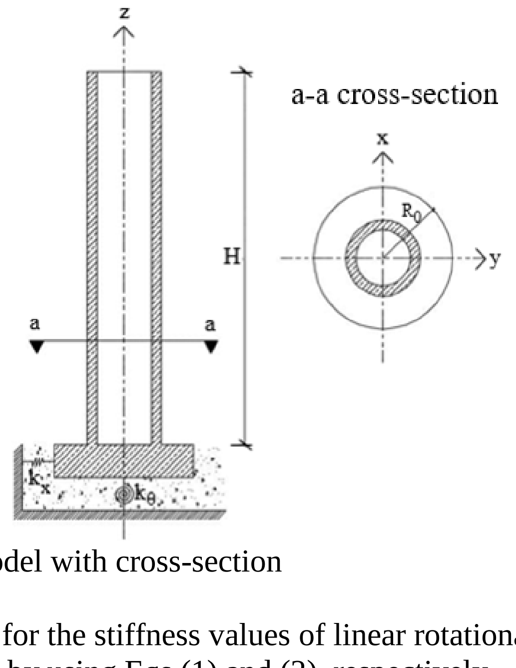

Figure 1. The chimney model with cross-section

ASCE 4-98 (1998) is used for the stiffness values of linear rotational and translational springs. k_x and k_θ can be calculated by using Eqs.(1) and (2), respectively.

$$k_x = \frac{32(1-\upsilon)G_s R_0}{7-8\upsilon} \quad (1)$$

$$k_\theta = \frac{8G_s R_0^3}{3(1-\upsilon)} \quad (2)$$

where, G_s is shear modulus of soil and υ is Poisson's ratio of soil.

The governing equation of motion of the chimney can be written according to Timoshenko beam theory (TBT) as follows:

$$\frac{AG}{\bar{k}}\left(\frac{\partial^2 w(z,t)}{\partial z^2} - \frac{\partial \theta(z,t)}{\partial z}\right) - \bar{m}\frac{\partial^2 w(z,t)}{\partial t^2} = 0 \quad (3)$$

$$EI\frac{\partial^2 \theta(z,t)}{\partial z^2} - \bar{m}\frac{I}{A}\frac{\partial^2 \theta(z,t)}{\partial t^2} + \frac{AG}{\bar{k}}\left(\frac{\partial w(z,t)}{\partial z} - \theta(z,t)\right) = 0 \quad (4)$$

Here; w(z,t) is lateral deflection function, $\theta(z,t)$ is rotation function, $\bar{m}$ is mass per unit length, A is cross-sectional area, G is shear modulus, E is Young's modulus, I is area moment of inertia and $\bar{k}$ is shear correction factor according to TBT.

Assuming the motion of chimney is harmonic and using separation of variables method, the w(z,t) and $\theta(z,t)$ can be rewritten as:

$$w(\xi,t) = w(\xi) \cdot e^{i\omega t} \quad (5)$$

$$\theta(\xi,t) = \theta(\xi) \cdot e^{i\omega t} \quad (6)$$

where $\xi = z / H$

Substituting Eqs. (5) and (6) into Eqs.(3) and (4), the following equations are obtained.

$$\frac{AG}{\bar{k}H^2}\frac{d^2 w}{d\xi^2} - \frac{AG}{\bar{k}H}\frac{d\theta}{d\xi} + \bar{m}\omega^2 w(\xi) = 0 \quad (7)$$

$$\frac{EI}{H^2}\frac{d^2\theta}{d\xi^2} + \frac{AG}{\bar{k}H}\frac{dw}{d\xi} + \left(\frac{\bar{m}I\omega^2}{A} - \frac{AG}{\bar{k}}\right)\theta(\xi) = 0 \quad (8)$$

where ω represents angular natural frequency.

The bending moment function and shear force function of chimney are written in Eqs. (9) and (10), respectively.

$$M(\xi) = \frac{EI}{H}\frac{d\theta}{d\xi} \quad (9)$$

$$T(\xi) = \frac{AG}{\bar{k}H}\frac{dw}{d\xi} - \frac{AG}{\bar{k}}\theta(\xi) \quad (10)$$

The boundary conditions of the chimney can be written as Eqs. (11)-(14):

$$M(\xi=0) + \theta(\xi=0)k_\theta = 0 \quad (11)$$

$$T(\xi=0) + w(\xi=0)k_x = 0 \quad (12)$$

$$M(\xi=1) = 0 \quad (13)$$

$$T(\xi=1) = 0 \quad (14)$$

The natural frequencies can be calculated by equating the determinant of the coefficient matrix to zero after obtaining the equation system using boundary conditions.

DYNAMIC STIFFNESS METHOD (DSM)

The dynamic stiffness matrix of chimney can be constructed by using displacements and forces at each end of member. The vector of end displacements of chimney and the vector of coefficients are given in Eqs. (15) - (16), respectively.

$$\delta = \begin{bmatrix} w_0 & \theta_0 & w_1 & \theta_1 \end{bmatrix} \tag{15}$$

$$\overline{C} = \begin{bmatrix} \overline{C}_1 & \overline{C}_2 & \overline{C}_3 & \overline{C}_4 \end{bmatrix} \tag{16}$$

where

$w_0 = w(\xi=0) + T(\xi=0)/k_x, \theta_0 = \theta(\xi=0) + M/(\xi=0)k_\theta, w_1 = w(\xi=1), \theta_1 = \theta(\xi=1)$

The matrix form of displacements can be written as closed form:

$$\delta = \Delta\overline{C} \tag{17}$$

where Δ is a 4x4 coefficient matrix that obtained from end displacement functions. The end forces of the beam is given in vector form in Eq. (18):

$$F = \begin{bmatrix} T_0 & M_0 & T_1 & M_1 \end{bmatrix}^T \tag{18}$$

where

$T_0 = T(\xi=0) + w(\xi=0)k_x, M_0 = M(\xi=0) + \theta(\xi=0)k_\theta, T_1 = T(\xi=1), M_1 = M(\xi=1)$

The matrix form of force functions is given in Eq. (19):

$$F = \kappa\overline{C} \tag{19}$$

where κ is a 4x4 coefficient matrix that obtained from end force functions.

Eqs.(18) and (19) are used to construct the dynamic stiffness matrix of the Timoshenko element chimney as:

$$F = \kappa\Delta^{-1}\delta \tag{20}$$

In Eq.(20), $\kappa\Delta^{-1}$ represents the dynamic stiffness matrix and natural frequency values are calculated by equating the determinant of $\kappa\Delta^{-1}$ to zero.

NUMERICAL ANALYSIS AND DISCUSSION

The numerical analyses are performed using the following data: H = 30 m, R_0 = 2.15 m, outer diameter of chimney = 2.30 m, inner diameter of chimney = 1.70 m, $\overline{m}$ = 4.8038 kNsn^2/m, E = 31000000 kN/m^2, G = 12916667 kN/m^2, $\overline{k}$ = 2, υ=0.25. The spring stiffness values that used in numerical analysis are presented in Table 1. The natural frequencies that calculated using DSM and analytical approach are given in Table 2 including fixed support results.

Table 1. Shear modulus values of soil and elastic spring constants

G_s (kN/m^2)	30000	60000	90000	120000	150000
k_x (kN/m)	309600	619200	928800	1238400	1548000
k_θ (kNm/rad)	1060093	2120187	3180280	4240373	5300467

Table 2. First five natural frequencies of Timoshenko element chimney

Method	Natural Frequency (Hz)	Soil Shear Modulus (kN/m^2)					Fixed Support Condition
		30000	60000	90000	120000	150000	
DSM	1st	**0.6950**	**0.8970**	**1.0165**	**1.1025**	**1.1609**	**1.5391**
Analytical		0.6950	0.8970	1.0165	1.1025	1.1609	1.5391
DSM	2nd	**6.5111**	**6.9756**	**7.2595**	**7.4847**	**7.6520**	**9.2392**
Analytical		6.5111	6.9756	7.2595	7.4847	7.6520	9.2392
DSM	3rd	**17.0726**	**18.8658**	**19.8174**	**20.3813**	**20.7632**	**24.3410**
Analytical		17.0726	18.8658	19.8174	20.3813	20.7632	24.3410
DSM	4th	**30.1008**	**33.5977**	**35.6047**	**36.8552**	**37.7131**	**44.2132**
Analytical		30.1008	33.5777	35.6047	36.8552	37.7131	44.2132
DSM	5th	**48.6260**	**51.4464**	**53.7719**	**55.5630**	**56.9314**	**67.3527**
Analytical		48.6260	51.4464	53.7719	55.5630	56.9314	67.3527

Table 2 shows that DSM solutions are in very good agreement with analytical solution. According to results, the error that arised from assumption of fixed support is increased for decreasing soil shear modulus for all modes. Figure 2 represents the relative error between fixed support assumption and SSI consideration. It seen from Figure 2 that, fixed support assumption cause higher errors for the fundamental mode in comparison with other modes. Moreover, it is revealed that the sensitivity of higher modes to SSI are very close for different soil shear modulus values.

The mode shapes are plotted by equating a nonzero displacement to an arbitrary value. The first five mode shapes of Timoshenko element chimney are presented in Figure 3.

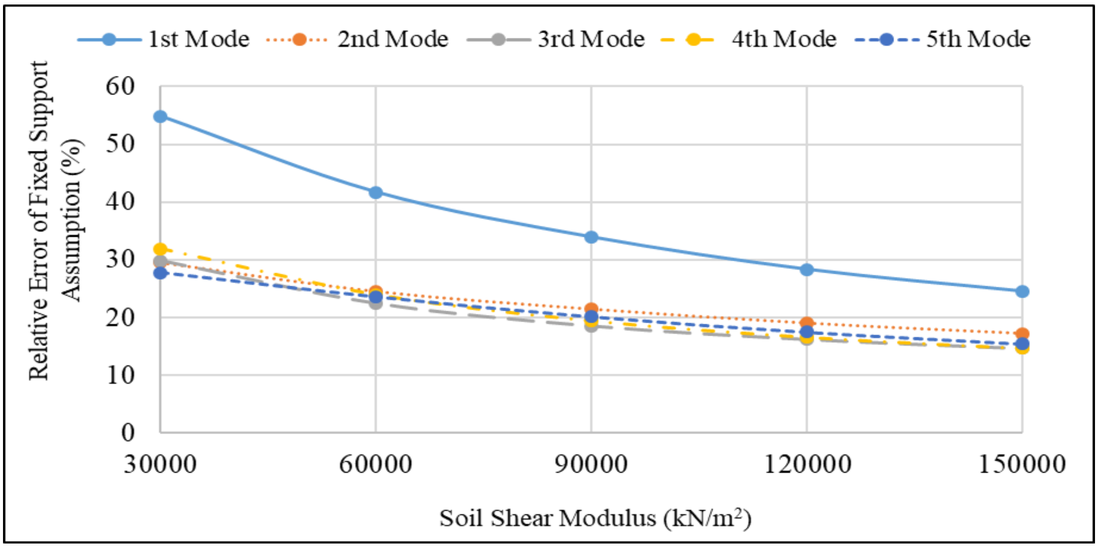

Figure 2. Relative error of fixed support assumption for different soil shear modulus values

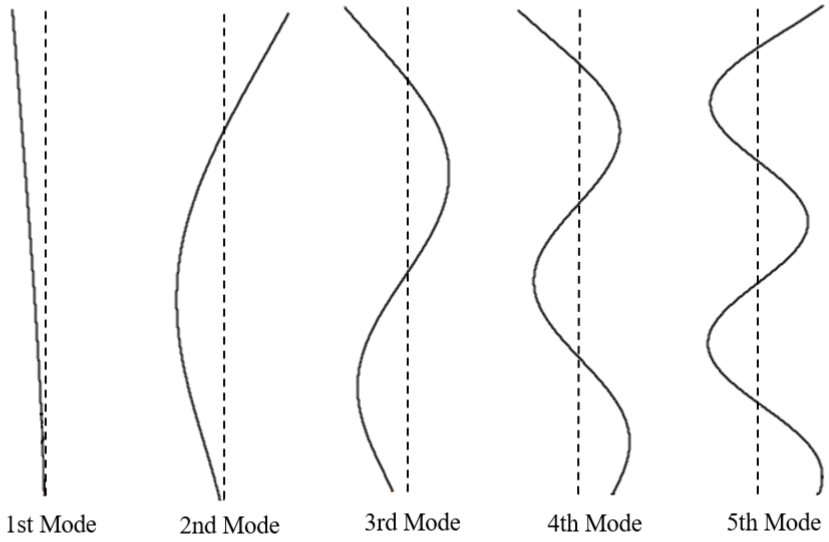

Figure 3. First five mode shapes of Timoshenko element chimney (G_S =150000 kN/m^2)

CONCLUSION

This study reveals the importance of considering SSI on the free vibrations of Timoshenko element chimneys using dynamic stiffness approach. The DSM results are in very good agreement with analytical results. The fixed support assumption in the dynamic analysis of chimney type structures may cause up to 55% error for weak soil. The computer programs that prepared for calculating natural frequencies and plotting mode shapes are working fast.

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